DHANALAKSHMI SRINIVASAN ENGINEERING COLLEGE, PERAMBALUR – 621212

DEPARTMENT OF ECE

QUESTION BANK

YEAR / SEM: II / IV **SUB. CODE: EC6403**

SUB. NAME: ELECTROMAGNETIC FIELDS

UNIT I – STATIC ELECTRIC FIELD

PART – A

1. ***State Divergence Theorem. (May / June 2016) (Apr / May 2013) (Nov / Dec 2012)***

The integral of the normal component of any vector field over a closed surface is equal to the integral of the divergence of this vector field throughout the volume enclosed that closed surface.

$$\iiint \nabla.A \; dv = \oiint A ds$$

2. ***Define Electric dipole. (May / June 2016)***

Dipole or electric dipole is two equal and opposite point charges are separated by a very small distance.

3. ***State Gauss law and write its applications (Nov / Dec 2015)***

The electric flux passing through any closed surface is equal to the total charge enclosed by that surface.

$$\chi = Q$$

4. ***State Coulombs law. (Nov / Dec 2016)***

The force of attraction or repulsion between any two point charges is directly proportional to the product of two charges and inversely proportional to the square of the distance between them.

$$F \propto \frac{Q_1 Q_2}{r^2}$$

5. ***What is an electric potential? Write expression for potential due to an electric dipole. (Nov / Dec 2016)***

It is the work done in moving a unit positive charge from one point to another in an electric field.

$$V = \frac{W}{Q} = -\int_B^A E dl$$

Dipole or electric dipole is two equal and opposite point charges are separated by a very small distance.

$$V = \frac{Q}{4\pi \varepsilon}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

6. State Stoke's theorem. (Apr / May 2015) (Nov / Dec 2014)

The line integral of a vector H around a closed path S is equal to the surface integral of the normal component of its curl over any closed surface.

$$\oint H\, dl = \iint \nabla \times H \; ds$$

7. What is the relationship between electric scalar potential and electric field intensity? (Apr / May 2015) (Nov / Dec 2013)

If two points are separated by an infinitesimal distance dr, the work done by an external force in moving a unit positive charge from one point to the other will be dV = – E.dr.

$$E = -\nabla V$$

8. What is Gradient? (Apr / May 2014) (Nov / Dec 2013)

The gradient of any scalar function is the maximum space rate of change of that function. If the scalar V represents the electric potential, ∇V represents potential gradient.

$$\nabla V = \frac{\partial V}{\partial x}\overline{a_x} + \frac{\partial V}{\partial y}\overline{a_y} + \frac{\partial V}{\partial z}\overline{a_z}$$

$$\nabla V = grad\ V$$

9. What is the significance of electric flux density? (Nov / Dec 2012)

It is defined as electric flux per unit area. It is denoted by D.

D = Q/A

10. Define Curl.

The curl of a vector A at any point is defined as the limit of its surface integral of its cross product with normal over a closed surface per unit volume shrinks to zero.

$$\nabla \times A = curl\ H = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_Z \end{vmatrix}$$

PART – B

1. ***Define the potential difference and electric field. Give the relation between potential and field intensity. Also derive an expression for potential due to infinite uniformly charged line and also derive potential due to electric dipole. (May / June 2016) (Apr / May 2015) (Apr / May 2014)***

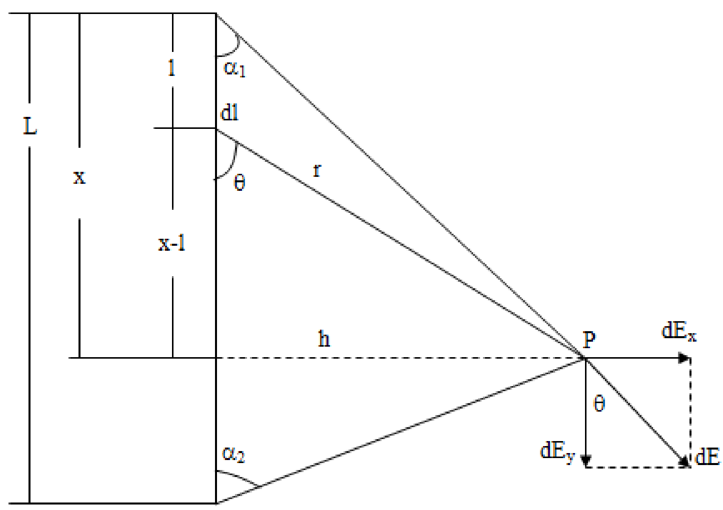
Electric field: It is defined as the electric force per unit charge.

$$E = \frac{Q}{4\pi\ \varepsilon r^2}$$

Potential Difference: It is the work done in moving a unit positive charge from one point to another in an electric field.

$$V = \frac{Q}{4\pi\ \varepsilon}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Line Charge – Diagram



Electric Field for x component: $E_x = \frac{\rho_l}{4\ \pi\ \varepsilon\ h}[\cos\alpha_1 + \cos\alpha_2]$

Electric Field for y component: $E_y = \frac{\rho_l}{4\ \pi\ \varepsilon\ h}[\sin\alpha_1 - \sin\alpha_2]$

Case 1: If point is at bisector of the line, i.e $\alpha_1 = \alpha_2 = \alpha$

$$E = \frac{\rho_l}{2\ \pi\ \varepsilon\ h}\cos\alpha$$

Case 2: if the line is infinitely long, then $\alpha = 0$.

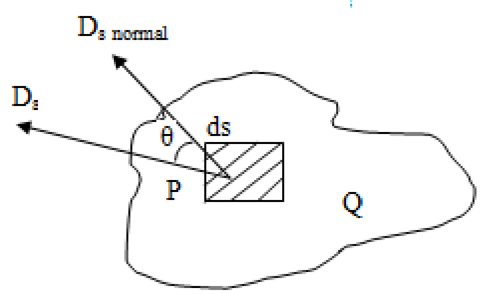
$$E = \frac{\rho_l}{2\pi\ \varepsilon h}$$

2. ***State and prove Gauss law and explain any one of the applications of Gauss law. (May / June 2016) (Apr / May 2015) (Nov / Dec 2013)***

Definition: The electric flux passing through any closed surface is equal to the total charge enclosed by the surface.

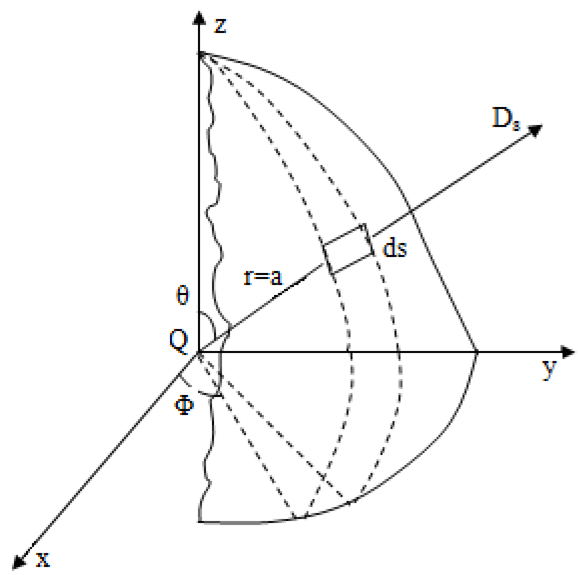
$$\chi = Q$$

Closed Surface Diagram:



$$\chi = \oint D_s . d\ s = \oint \rho_v . d\ v = Q$$

Spherical Coordinate System Diagram:



Proof:

$$E = \frac{Q}{4\ \pi\ \varepsilon r^2} \ \&\ D = \frac{Q}{4\ \pi\ r^2}$$

Gauss law in integral form:

For Line charge: $Q = \int \rho_l d\ l$

For Surface charge: $Q = \iint \rho_s . d\ s$

For volume charge: $Q = \iiint \rho_v . d\ v$

Applications of Gauss law:

i) Electric field intensity due to infinite sheet of charge:

$$\Psi = \iint_{top} D.ds + \iint_{bottom} D.ds + \iint_{sides} D.ds$$

$$E = \sigma/2\varepsilon$$

3. ***State and explain Divergence theorem. (Nov / Dec 2015) (Nov / Dec 2013)***

Theorem: The volume integral of the divergence of a vector field over a volume is equal to the surface integral of the normal component of this vector over the surface bounding the volume.

$$\iiint_v \nabla.A \, dv = \oiint A ds$$

Proof:

$$\iiint_v \nabla.A \, dv = \iiint_v \left[\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right] dx\, dy\, dz$$

Considering one element (x or y or z) in volume,

$$\iiint_v \frac{\partial A_x}{\partial x} dx\, dy\, dz = \oiint A_x . ds_x$$

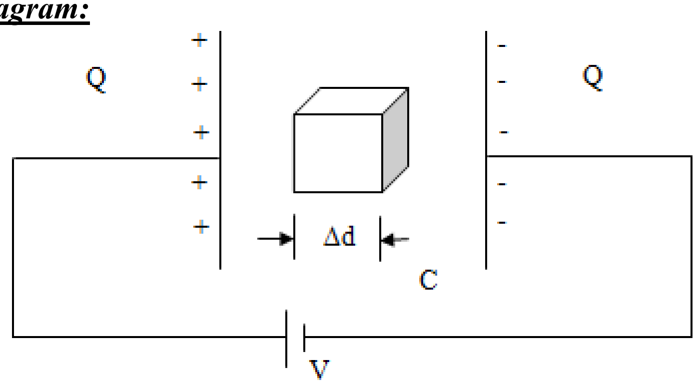
4. ***i) Derive the expression for energy stored in an electrostatic field in terms of field quantities. (Nov / Dec 2015)***

Electrostatic energy: The capacitor stores the electrostatic energy equal to work done to build up the charge.

$$W = \int_0^Q \frac{Q}{C} dQ = \frac{Q^2}{2C}$$

$$W = \frac{1}{2} Q V$$

Energy Density Diagram:



Capacitance: $\Delta C = \varepsilon \Delta d$

Energy stored: $\Delta W = \frac{1}{2} \Delta C (\Delta V)^2$

Potential difference: $\Delta V = E.\Delta d$

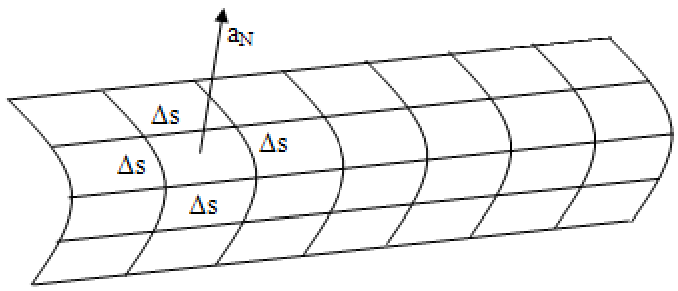
Energy density is: $\Delta W/\Delta v = \frac{1}{2} D.E$

ii) State and prove Stoke's theorem. (Nov / Dec 2016) (Nov / Dec 2013).

Theorem: The line integral of a vector around a closed path is equal to the surface integral of the normal component of its curl over any closed surface.

$$\oint H d\, l = \iint \nabla \times H \ d\, s$$

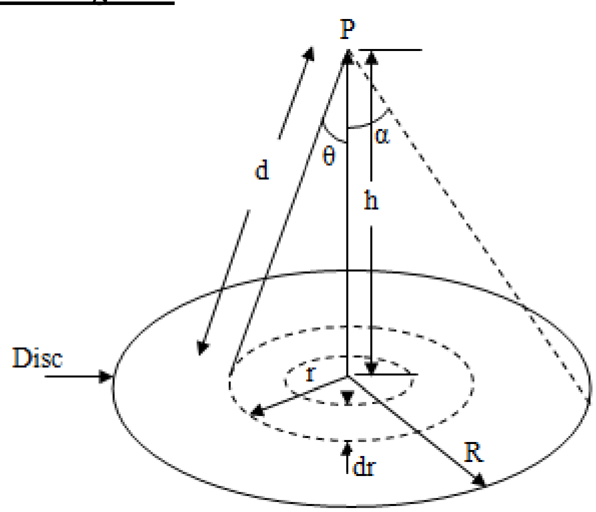
Stoke's Theorem Diagram:



$$\oint \frac{H.d\,\Delta s}{\Delta s} = (\nabla \times H) a_N$$

5. ***A circular disc of radius 'a' meter is charged uniformly with a charge of ρ c/m. Find the electric field intensity at a point h meter from the disc along its axis. (Nov / Dec 2016) (Apr / May 2015) (Apr / May 2014)***

   Charged Circular disc Diagram:



$$d\ E = \frac{\rho_s . d\ s}{4\pi\ \varepsilon\ d^2}$$

$$d\ E_y = \frac{\rho_s . d\ s}{4\pi\ \varepsilon\ d^2} c\ o\ s\ \theta$$

$$d\ E_y = \frac{\rho_s . s\ i\ n\ \theta}{2\varepsilon}$$

The total electric field over a charged disc is $E = \frac{\rho_s}{2\ \varepsilon}\left[1 - \frac{h}{\sqrt{h^2 + R^2}}\right]$

UNIT II – CONDUCTORS AND DIELECTRICS

PART – A

1. ***Write the Laplace's equation in all the three conditions. (May / June 2016)***

   For Cartesian coordinate system: $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$

   For Cylindrical coordinate system: $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial v}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 v}{\partial \emptyset^2}\right) + \frac{\partial^2 v}{\partial z^2} = 0$

   For Spherical coordinate system: $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 v}{\partial \emptyset^2} = 0$

2. ***What is dielectric polarization? (May / June 2016)***

   Polarization is defined as dipole moment / unit volume.

   $$P = \mathrm{Lt}_{\Delta v \to 0} \frac{1}{\Delta v}\sum_{i=1}^{n\Delta v} P_i \ \text{c/m}^3.$$

3. ***What are the boundary conditions for electric field at the perfect dielectric conductor interface? (Nov / Dec 2015) (Apr / May 2014)***
   a) The field intensity inside a conductor is zero and the flux density inside a conductor is zero.
   b) No charge can exist within a conductor.
   c) The charge density within the conductor is zero.

4. ***Find the energy stored in the 20pF parallel plate capacitor with plate separation of 20cm. the magnitude of electric field in the capacitor is 1000 V/m. (Nov / Dec 2015)***

   Given: C = 20pF; d = 20cm; E = 1000 V/m

   $$V = E.d$$
   $$W = \tfrac{1}{2} CV^2$$
   $$V = 200$$
   $$W = 2 \times 10^{-9} \text{ Joules}$$

5. ***Derive resistance of a conductor. (Nov / Dec 2016)***

   Resistance of the conductor is given by R = ρl / A

   ρ = Resistivity of the material   l = length of the conductor

   A = Area of the cross section of the conductor.

6. ***Give Poisson's and Laplace equation. (Nov / Dec 2016) (Apr / May 2015)***

   Poisson' equation = $\nabla^2 V = -\frac{\rho}{\varepsilon}$

Laplace equation = $\nabla^2 V = 0$.

7. ***What is displacement current? (Apr / May 2015)***

The displacement current I_D is flowing through a capacitor when ac voltage is applied across the capacitor. $I_D = dQ / dt$.

8. ***State point form of ohm's law. (Apr / May 2013) (Nov / Dec 2014)***

The electric field strength within a conductor is proportional to the current density.

$$J \propto E$$

$$J = \sigma E$$

9. ***Distinguish between conduction current and displacement current. (Apr / May 2013)***

Conduction Current: The conduction current I_C is flowing through a conductor whose resistance is R when potential V is applied across the conductor. $I_C = V / R$.

Displacement Current: The displacement current I_D is flowing through a capacitor when ac voltage is applied across the capacitor. $I_D = dQ / dt$.

10. ***Write the equation of continuity. (Nov / Dec 2013) (Nov / Dec 2012)***

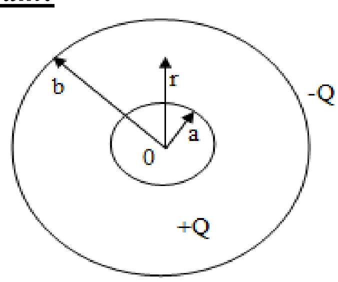
Continuity Equation in Integral form: $\oint J . d\, s = -\frac{d\, Q}{d\, t}$

Continuity Equation in point form: $\nabla . J = \frac{\partial\, \rho}{\partial\, t}$

<u>PART – B</u>

1. ***Derive the capacitance of a spherical capacitor. (May / June 2016) (Nov / Dec 2015)***

<u>Spherical Capacitance Diagram:</u>



The electric field intensity at any point between inner and outer sphere is given by:

$$E = -\int_b^a \frac{Q}{4\pi\, \varepsilon r^2} \,(a \leq r \leq b)$$

The potential difference between the spheres is

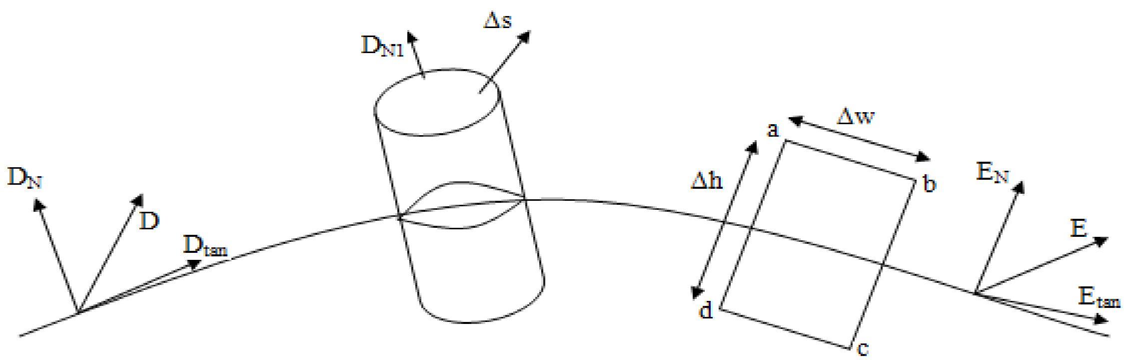
$$V = -\int_b^a \frac{Q}{4\pi\varepsilon r^2}\, dr$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{b-a}{ab}\right]$$

The capacitance of a spherical is $C = \frac{Q}{V} = 4\pi\varepsilon\left[\frac{ab}{b-a}\right]$

2. ***Derive the boundary conditions of the tangential and normal components of electric field at the interface of two mediums with dielectrics. (May / June 2016) (Nov / Dec 2016) (Apr / May 2013)***

Boundary Diagram:



$$\oint E\,dl = 0$$

$$\oint_a^b E\,dl + \oint_b^c E\,dl + \oint_c^d E\,dl + \oint_d^a E\,dl = 0$$

$$E_1 = E_{1t} + E_{1N}$$

$$E_2 = E_{2t} + E_{2N}$$

$$E_{tan1}(\Delta W) = E_{tan2}(\Delta W)$$

$$\frac{D_{tan1}}{D_{tan2}} = \frac{\varepsilon_1}{\varepsilon_2}$$

The boundaries are:

i) $E_{tan1} = E_{tan2}$

ii) $\frac{D_{tan1}}{D_{tan2}} = \frac{\varepsilon_1}{\varepsilon_2}$

iii) $D_{N1} - D_{N2} = \rho_s$

iv) $\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1}$

3. ***Derive the expression for relaxation time by solving the continuity equation. (Nov / Dec 2015)***

   The current through the closed surface is $I = \oint J.d\,s$

$$I = \oint J d\,s = -\frac{d\,Q}{d\,t}$$

   <u>Integral form</u> of continuity equation of current: $\oint J.d\,s = -\frac{d\,Q}{d\,t}$

$$\int J d\,s = \int (\nabla.J) d\,v$$

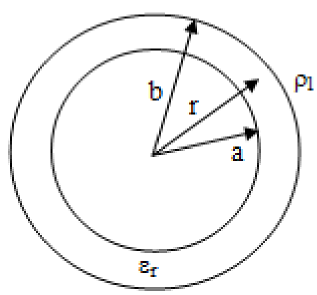
   <u>Differential or point form</u> of continuity equation: $\nabla.J = \frac{\partial\,\rho_v}{\partial\,t}$

4. ***Derive the expression for capacitance of a coaxial cable. (Nov / Dec 2016)***

   Considering a coaxial cable of inner radius 'a' and outer radius 'b', the relative permittivity of dielectrics filled in between two co-axial cylinders is ε_r. By applying gauss law, the electric field E at any distance from the axis of cylinders is

$$E = \frac{\rho_l}{2\pi\,\varepsilon\,r}$$

   <u>Co-axial cable Diagram:</u>



   The potential difference between two coaxial cable is $V = -\int_b^a E.d\,r$

$$V = -\frac{\rho_l}{2\pi\,\varepsilon} l\,n\left(\frac{b}{a}\right)$$

   The capacitance on coaxial cable is $C = \frac{\rho_l}{V} = \frac{2\,\pi\,\varepsilon}{l\,n\left({}^{b}/_{a}\right)}$

5. ***Derive the capacitance of a parallel plate capacitor. (Nov / Dec 2014)***

   Considering a parallel plate capacitor consists of two dielectrics, the relative permittivity of dielectric medium 1 and 2 are ε_{r1} and ε_{r2} respectively.

$$V_1 = E_1 d_1$$

$$V_2 = E_2\,(d - d_1), \text{ where } d_2 = d - d_1$$

$$V = E_1 d_1 + E_2\,(d - d_1)$$

w.k.t D = εE

$$\therefore V = \frac{Q}{A\,\varepsilon_0}\left[\frac{d_1}{\varepsilon_{r1}} + \frac{(d-d_1)}{\varepsilon_{r2}}\right]$$

The capacitance on parallel plates is $C = \frac{A\,\varepsilon_0\varepsilon_r}{d_1\varepsilon_r + (d-d_1)}$ if $\varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_r$

6. Write the Poisson's and Laplace's equations. (May / June 2010)

<u>Poisson's Equation:</u> According to the Gauss law in point form, the divergence of electric flux density is equal to the volume charge density.

$$\nabla . D = \rho_v$$

w.k D = εE

$$\nabla . E = \frac{\rho_v}{\varepsilon}$$

Similarly $E = -\nabla V$

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

For Cartesian coordinate system: $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\varepsilon}$

For Cylindrical coordinate system: $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial v}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 v}{\partial \phi^2}\right) + \frac{\partial^2 v}{\partial z^2} = -\frac{\rho_v}{\varepsilon}$

For Spherical coordinate system: $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v}{\partial \phi^2} = -\frac{\rho_v}{\varepsilon}$

<u>Laplace Equation:</u> If the volume charge density (ρ_v) is zero.

$$\nabla^2 V = 0$$

For Cartesian coordinate system: $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$

For Cylindrical coordinate system: $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial v}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 v}{\partial \phi^2}\right) + \frac{\partial^2 v}{\partial z^2} = 0$

For Spherical coordinate system: $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v}{\partial \phi^2} = 0$

UNIT III – STATIC MAGNETIC FIELDS

PART – A

1. ***Define magnetic vector and scalar potential. (May / June 2016) (Nov / Dec 2016) (Apr / May 2015) (Apr / May 2013) (Nov / Dec 2014)***

Magnetic Vector Potential: It is defined as that quantity whose curl gives the magnetic flux density. $B = \nabla \times A$.

Magnetic Scalar Potential: It is defined as that quantity whose negative gradient gives the magnetic intensity if there is no current source present. $H = -\nabla V_m$.

2. ***A current of 3A flowing through in an inductor of 100mH. What is the energy stored in inductor? (May / June 2016)***

Given: L = 100mH; I = 3A

$$W = \frac{1}{2} LI^2.$$

$$W = 450 \times 10^{-3} \text{ Joules.}$$

3. ***State Biot Savart Law. (Nov / Dec 2015) (Apr / May 2014) (Apr / May 2013)***

It states that the magnetic flux density at any point due to current element is proportional to the current element and inversely proportional to the square of the distance between them.

$$dB = \frac{\mu I\, dl}{4\pi r^2} \sin\theta$$

4. ***Derive point form of Ampere Circuital Law. (Nov / Dec 2015) (Nov / Dec 2016)***

Magnetic field intensity around a cloed path is equal to the current enclosed by that path.

$$\int H.dI = I$$

5. ***Define the term magnetic flux density. (Nov / Dec 2012)***

It is defined as the magnetic flux passing per unit area. $B = \Phi / A$ (Tesla).

6. ***List the applications of Ampere's Circuital law.***

It involves in finding the total current enclosed by a closed path. Ex: It is used to find the magnetic field intensity due to infinitely long straight conductor and coaxial cable.

7. ***What is solenoid?***

An inductor coil consisting of multiple turns of wire wound in a helical geometry around a cylindrical core is called as solenoid.

8. ***A round copper conductor is carrying a current of 250A. Determine the magnetizing force and flux density at a distance of 10cm from the conductor.***

   Soln: Flux Density, $B = \frac{\mu_0 N I}{2 \pi r}$; B = 0.5 mWb/m^2

   Magnetic force, H = B / μ_0; H = 397.88 AT/m.

9. ***A circular coil of radius 10cm is made up of 100 turns. It carries a current of 5A. Compute the magnetic field intensity at the centre of the coil.***

   H = NI / 2a; H = 2500 AT/m

10. ***What is the magnetic field intensity due to toroid and solenoid?***

    Toroid: H = NI / 2πr

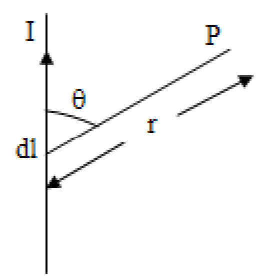
    Solenoid: H = NI / l

PART – B

1. ***State Biot Savart law. Derive the expression for magnetic field intensity and magnetic flux density at the centre of the square current loop of side l.(May / June 2016) (Nov / Dec 2016)***

Biot – Savart Law: The magnetic flux density produced by a current element at any point in a magnetic field is proportional to the current element and inversely proportional to the square of the distance between them.

Biot Savart - Diagram:



The magnetic flux density: $d\ B = \frac{\mu\ I\ d\ l}{4\ \pi\ r^2}\bar{a}$

Its magnitude is $d\ B = \frac{\mu\ I\ d\ l\ s\ i\ n\ \theta}{4\ \pi\ r^2}$

The magnetic field intensity: $d\ H = \frac{I\ d\ l}{4\ \pi\ r^2}\bar{a}$

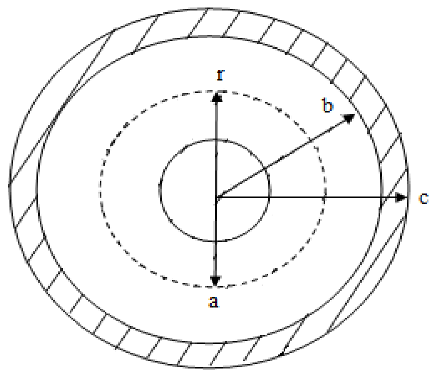
Its magnitude is $d\ H = \frac{I\ d\ l\ s\ i\ n\ \theta}{4\ \pi\ r^2}$.

2. ***Derive an expression for magnetic field due to an infinitely long coaxial cable. (May / June 2016) (Apr / May 2013)***

   Infinitely long coaxial cable:

Consider an infinitely long transmission line consisting of two concentric cylinders having their axes along the z-axis.

<u>Coaxial Cable Diagram:</u>



i) The field intensity within the solid inner conductor of radius ($0 \leq r \leq a$)

$$H = \frac{I\, r}{2\pi\, a^2}$$

ii) The field intensity in between inner and outer conductor ($a \leq r \leq b$)

$$H = \frac{I}{2\pi\, r}$$

iii) The field intensity in the annular (outer conductor) ($b < r < c$)

$$H_\emptyset = \frac{I}{2\pi\, r}\left[1 - \frac{(r^2 - b^2)}{(c^2 - b^2)}\right]$$

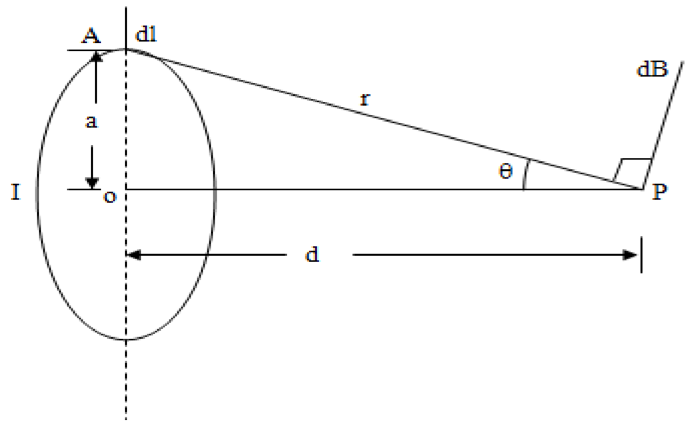
iv) The field intensity outside the cable $r \geq c$

$H_\emptyset = 0.$

3. ***An infinite long straight conductor with a circular cross section of radius 'b' carries a steady current I. Determine magnetic flux density both inside and outside conductor. (Nov / Dec 2015) (Apr / May 2014) (Nov / Dec 2013)***

Consider a circular coil of radius 'b' carrying a current 'I' and also consider a current element Idl.

<u>Circular coil carrying current I - Diagram:</u>



$$d\,B = \frac{\mu\, I\, d\, l\ \ s\, i\, n\, \theta}{4\pi\ r^2}$$

$$B = \frac{\mu_0 a^2 I}{2(a^2 + d^2)^{3/2}}$$

The magnetic field intensity is H= $\frac{a^2 I}{2(a^2 + d^2)^{3/2}}$

If d = 0, the field at the centre is $B = \frac{\mu_0 a^2 I}{2\, a^3}$

The magnetic field intensity is H= $\frac{I}{2\, a}$

4. ***Derive the expression for magnetic vector potential in terms of current density. (Nov / Dec 2015) (May / June 2010)***

If current is enclosed, the potential which depends upon current element is no more scalar but it is vector quantity. Since the divergence of a vector is a scalar, vector potential is expressed in curl.

$$\nabla . B = 0$$

$$B = \nabla \times A$$

Take curl on both sides, $\nabla \times B = \nabla \times \nabla \times A$

By the Identity, $\nabla \times \nabla \times A = \nabla(\nabla . A) - \nabla^2 A$

$$A_x = \frac{\mu}{4\pi} \int_v \frac{J_x}{r} d\ v$$

$$A_y = \frac{\mu}{4\pi} \int_v \frac{J_y}{r} d\ v$$

$$A_z = \frac{\mu}{4\pi}\int_v \frac{J_z}{r} d\ v$$

The magnetic vector potential is $A = \frac{\mu}{4\ \pi}\int_v \frac{J}{r} d\ v$

5. ***State Ampere's circuital law and discuss about any two simple applications of it. (Apr / May 2015) (Apr / May 2014) (Nov / Dec 2014)***

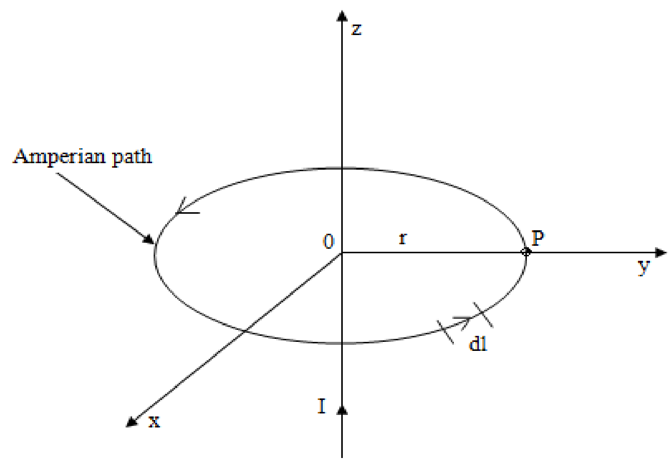
Ampere's Circuital Law: It states that the line integral of magnetic field intensity H about any closed path is exactly equal to the direct current enclosed by that path.

$$\oint H d\ l = I$$

Applications of Ampere's Circuital Law:

a) ***Infinite Line current:***

Infinite filamentary line current - Diagram:



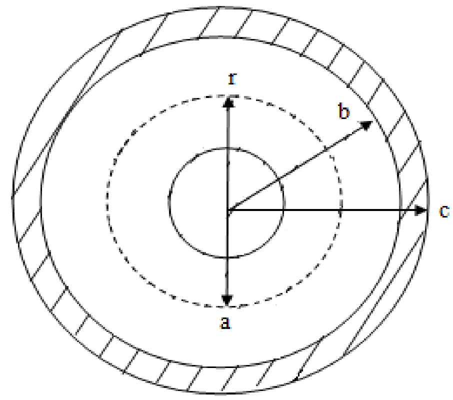
$$I = \oint H d\ l = H(2\pi\ r)$$

$$H = \frac{I}{2\pi\ r}\overline{a_r}$$

b) ***Infinitely long coaxial cable:***

Consider an infinitely long transmission line consisting of two concentric cylinders having their axes along the z-axis.

Coaxial Cable Diagram:



i) The field intensity within the solid inner conductor of radius ($0 \le r \le a$)

$$H = \frac{I\ r}{2\pi\ a^2}$$

ii) The field intensity in between inner and outer conductor ($a \le r \le b$)

$$H = \frac{I}{2\pi\ r}$$

iii) The field intensity in the annular (outer conductor) ($b < r < c$)

$$H_\phi = \frac{I}{2\pi\ r}\left[1 - \frac{(r^2 - b^2)}{(c^2 - b^2)}\right]$$

iv) The field intensity outside the cable $r \ge c$

$$H_\phi = 0$$

UNIT IV – MAGNETIC FORCES AND MATERIALS

PART – A

1. ***Mention the force between two current elements. (May / June 2016)***

   Force of attraction: $F = \frac{\mu_0 I_1 I_2}{2\,\pi\,d}$

   Force of repulsion: $F = \frac{\mu_0 I_1 I_2}{2\,\pi\,d}$. The magnitude of force is same.

2. ***Differentiate diamagnetic, paramagnetic and ferromagnetic material. (May / June 2016)***

   Diamagnetic: The metals and other elements having slight magnetic properties are called diamagnetic materials in which the magnetization is opposite to the applied field.

   Paramagnetic: If the magnetization is in the same direction as the applied field, such materials are called as paramagnetic materials.

   Ferromagnetic: It show very strong magnetic effects. The magnetization is in the same direction as the field.

3. ***What is the energy stored in a magnetic field in terms of field quantities. (Nov / Dec 2015)***

   $W = ½ LI^2$. L = Inductance & I = Current.

4. ***Calculate the mutual inductance of two inductively tightly coupled coils with self inductance of 25mH and 100mH. (Nov / Dec 2016)***

   L1 = 25mH & L2 = 100mH, K = 1

   $$M = \sqrt{L_1 L_2}$$

   M = 50mH

5. ***Give the expression for Lorentz force equation. (Nov / Dec 2016) (Nov / Dec 2013) (Apr / May 2015) (Nov / Dec 2012)***

   The force on a moving particle due to combined electric and magnetic field is given as

   F = Q (E + (v x B))

6. ***What is mutual inductance? (Apr / May 2015)***

   It is defined as the ratio of induced magnetic flux linkage in one coil to the current through the other coil.

   $$M = \frac{N_2 \emptyset_{1\,2}}{i_1}$$

7. ***What is magnetic dipole moment? (Nov / Dec 2013)***

It is defined as the maximum torque on loop per magnetic induction.

$$m = T / B$$

$$m = IA$$

8. ***Compare Self inductance and mutual inductance. (Nov / Dec 2013)***

Self Inductance: It is defined as the ratio of total magnetic flux linkage with the circuit to the current through the coil.

$$L = \Phi / i$$

Mutual Inductance: It is defined as the ratio of induced magnetic flux linkage in one coil to the current through the other coil.

$$M = \frac{N_2 \emptyset_{1\,2}}{i_1}$$

9. ***Define the term relative permeability. (Nov / Dec 2012)***

It is the measure of the ability of a material to support the formation of a magnetic field within itself.

$$\mu_0 = 4\pi \times 10^{-7}.$$

10. ***Give torque equation on closed circuits.***

The torque on closed circuit in a magnetic field if $T = BIA \sin\theta$ or $T = m \times B$ (vector form).

<u>PART – B</u>

1. ***Derive the expression for force on a moving charge in a magnetic field and Lorentz force equation. (May / June 2016) (Nov / Dec 2014)***

In an electric field, the force acting on a charged particle is $F_E = QE$.

The force is proportional to the product of the magnitude of charge Q, velocity V and flux density B, and the sine of the angle between them (v x B).

$$F_M = QvB\sin\theta$$

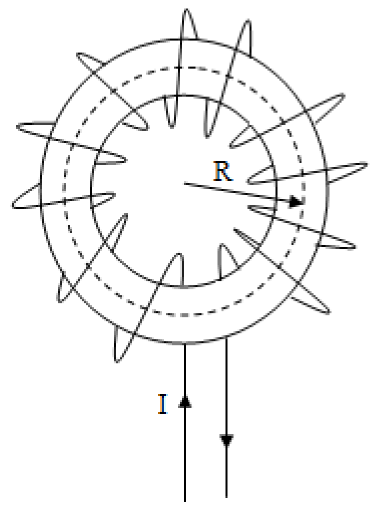
The force on a moving particle due to combined electric and magnetic field is

$$F = F_E + F_M$$

$F = Q (E + (v \times B))$. This force is called as Lorentz force.

2. ***Derive the inductance on a toroid. (May / June 2016)***

<u>Toroid Diagram:</u>



Consider a toroid of N number of turns carrying current I with mean radius R,

$$B = \frac{\mu_0 N\, I}{l}$$

$$l = 2\pi R$$

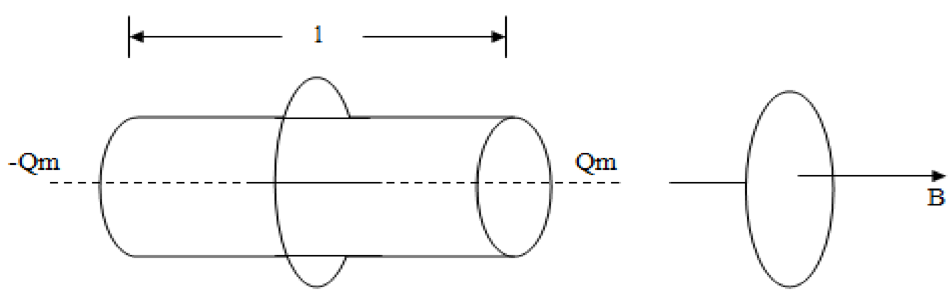
The flux linkage in the toroid is $N\phi$, i.e. $N\phi = NBA$

Inductance of the toroid is $L = \frac{N\emptyset}{I} = \frac{\mu_0 N^2 r^2}{2\ R}$

3. ***Explain about magnetization vector and derive the expression for relative permeability. (Nov / Dec 2015)***

Magnetization: The field produced due to movement of bound charges is called magnetization.

<u>Dipole Moment Diagram:</u>



Let the bound current I_b flows through a closed path. Assume that this closed path encloses differential area ds. Then the magnetic dipole moment is given by

$$m = I_b.ds$$

The magnetization is defined as the magnetic dipole moment per unit volume.
For a differential volume ΔV,

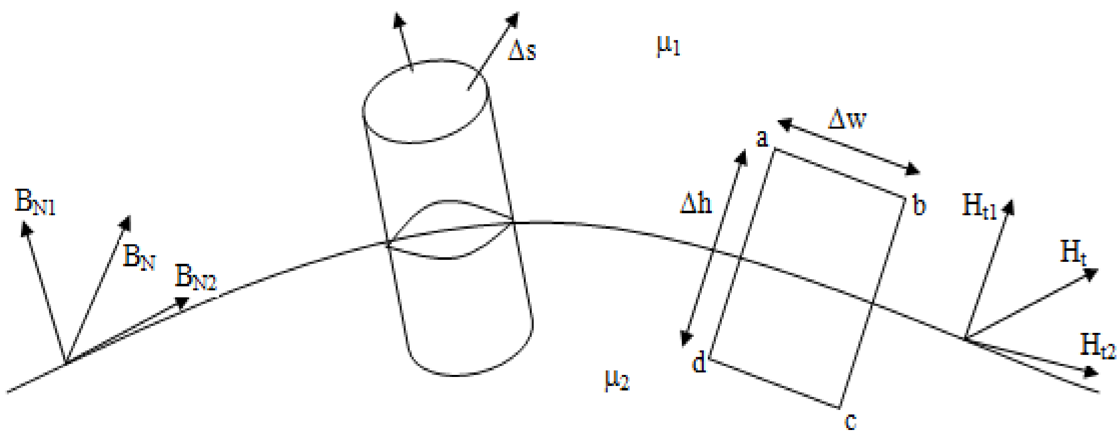
$$M_{total} = \underset{\Delta v \to 0}{\mathrm{Lt}} \frac{1}{\Delta v} \sum_{a=1}^{n\Delta v} m_a$$

4. ***Derive the boundary conditions of static magnetic field at the interface of two different magnetic medium. (Nov / Dec 2015) (Nov / Dec 2016)***

Condition: i) the tangential component of magnetic field intensity is continuous across the boundary

ii) The normal component of magnetic flux density is continuous across the boundary.

Boundary Diagram:



Applying Gauss law to the magnetic field

$$\oint B d s = 0$$

$$\oint B d s = \oint_{top} B d s + \oint_{lateral} B . d s + \oint_{bottom} B . d s = 0$$

$$\oint_{top} B d s = B_{N1} \Delta s$$

$$\oint_{lateral} B . d s = 0$$

$$\oint_{bottom} B . d s = -B_{N1} \Delta s$$

$$\therefore B_{N1} = B_{N2}$$

$$\text{w.k} \oint H . d l = I$$

$$H_{t1} - H_{t2} = K$$

$$K = \frac{B_{t\,1}}{\mu_1} - \frac{B_{t\,2}}{\mu_2}$$

5. ***Derive the expressions for inductance and magnetic flux density inside the solenoid. Calculate the inductance of the solenoid and energy stored when a current of 8A flowing through the solenoid of 2m long, 10 cm diameter and 4000 turns. (Nov / Dec 2016)***

Consider a solenoid of N number of turns carrying the current I. If B is the flux density and A is the area of cross section of the solenoid, then flux linkage through the solenoid is $N\phi = NBA$.

$$L = NBA / I$$

For long solenoid, $B = \mu_0 NI / l$

$$L = \mu_0 N^2 A / l.$$

Problem: to find $L = \mu_0 N^2 A / l$ & $W = \frac{1}{2} LI^2$.

$$A = \pi d^2 / 4$$

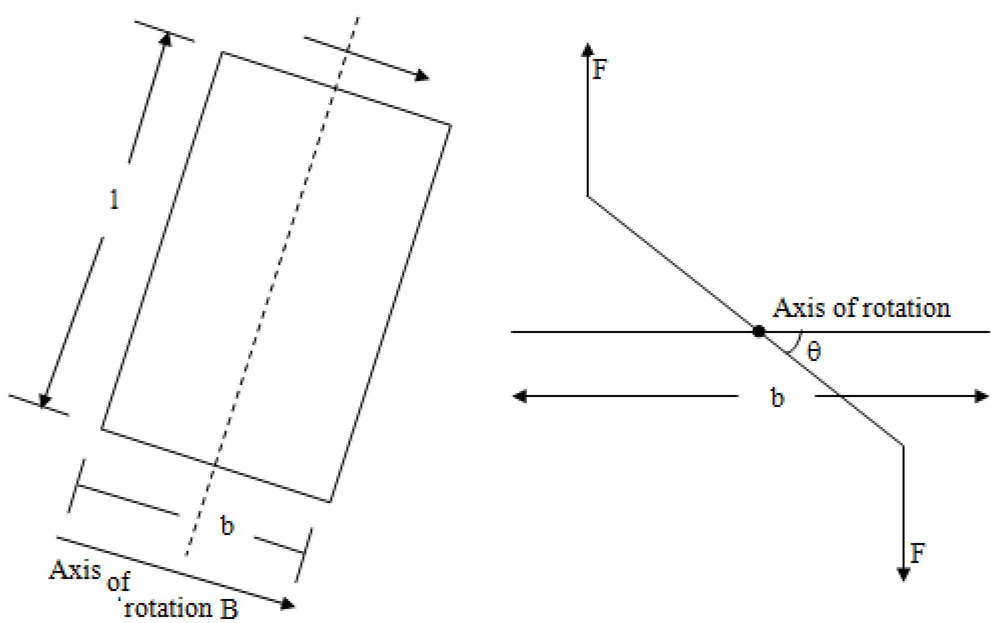
$$A = 0.785 \times 10^{-2}$$

$$L = 78.92\text{mH}$$

$$W = 25.2544 \text{ Joules.}$$

6. ***Derive an expression for torque on a loop carrying a current I. (Apr / May 2015) (Nov / Dec 2014) (Nov / Dec 2013)***

<u>Current Loop Diagram:</u>



When a current loop is placed parallel to a magnetic field, force act on the loop that tend to rotate it. The tangential force multiplied by the radial distance at which it acts is called Torque or mechanical moments on the loop.

Force on the loop, F = BIl sinθ

The total torque, T = 2 x torque on each side

$$T = BIA \sin\theta$$

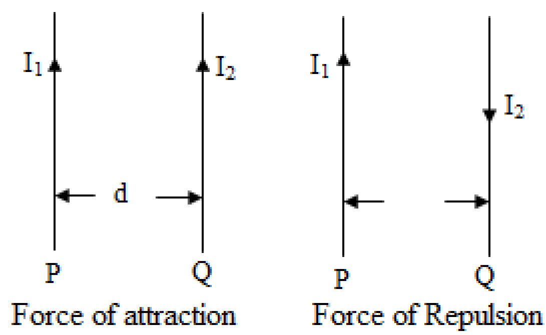
The magnetic moment of loop is IA, i.e. m = IA

Therefore, T = Bmsinθ

In vector form, T = m x B or m = T/B.

7. ***Derive an expression for force between two current carrying conductors. (Apr / May 2013)***

Diagram:



For finite (or) infinite length conductor, $B = \frac{\mu_0 I}{2 \pi d}$

Force of attraction: $F = \frac{\mu_0 I_1 I_2}{2 \pi d}$

Force of repulsion: $F = \frac{\mu_0 I_1 I_2}{2 \pi d}$. The magnitude of force is same.

UNIT V – TIME VARYING FIELDS AND MAXWELLS EQUATIONS

PART – A

1. ***State Faraday's law of induction. (May / June 2016) (Nov / Dec 2013) (Nov / Dec 2012).***

It states that electromagnetic force induced in a circuit is equal to the rate of change of magnetic flux linking the circuit.

emf = $d\Phi / dt$.

2. ***What is Poynting vector? (May / June 2016) (Apr / May 2015) (Nov / Dec 2014)***

The complex Poynting vector is $P = ½\, E \times H^*$ where H^* is complex conjugate of H.

3. ***What are the Maxwell's equations for free space medium? (Nov / Dec 2015)***

$$\oint H d l = \iint \frac{\partial D}{\partial t} . d s$$

$$\oint E d l = -\iint \frac{\partial B}{\partial t} . d s$$

$$\oiint D d s = 0$$

$$\oiint B d s = 0$$

4. ***Define Phase velocity. (Nov / Dec 2016)***

The phase velocity of a wave is the rate at which the phase of the wave propagates in space. This is the velocity at which the phase of any one frequency component of the wave travels. For such a component, any given phase of the wave (for example, the crest) will appear to travel at the phase velocity. The phase velocity is given in terms of the wavelength λ (lambda) and period T as

$$v_p = \lambda/T$$

5. ***What is skin effect? (Apr / May 2015) (Apr / May 2014)***

Skin effect is the tendency of an alternating electric current (AC) to become distributed within a conductor such that the current density is largest near the surface of the conductor, and decreases with greater depths in the conductor. The electric current flows mainly at the "skin" of the conductor, between the outer surface and a level called the skin depth.

$$\delta = \sqrt{\frac{2\rho}{\omega\ \mu}}$$

6. ***What is Brewster angle? (Apr / May 2015) (Apr / May 2013) (Nov / Dec 2014) (Nov / Dec 2013)***

Brewster's angle (also known as the polarization angle) is an angle of incidence at which light with a particular polarization is perfectly transmitted through a transparent dielectric surface, with no reflection. When polarized light is incident at this angle, the light that is reflected from the surface is therefore perfectly polarized.

$$r_p = \frac{\tan(\theta_1 - \theta_T)}{\tan(\theta_1 + \theta_T)}$$

7. ***Write down the expression for instantaneous and complex Poynting vector. (Apr / May 2013)***

Instantaneous power: $w = |V|\,|I| \cos(\omega t+\theta_v) \cos(\omega t+\theta_i)$

Instantaneous Poynting vector: P = E x H.

8. ***Maxwell's second equation is based on a famous law. What is it? Substantiate. (Nov / Dec 2014)***

Maxwell's second equation is based on Faraday's law which states that electromagnetic force induced in a circuit is equal to the rate of change of magnetic flux linking the circuit.

$$emf = d\Phi / dt$$

9. ***What is Uniform plane wave? (Nov / Dec 2014)***

If the phase of a wave is the same for all points on a plane surface, it is called plane wave. If the amplitude is also constant in a plane wave, it is called uniform plane wave.

10. ***Find the velocity of the uniform plane electromagnetic wave in free space.***

$$v_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

11. ***State Slepian vector.***

It is a vector which is defined at every point such that its flux coming out of any volume is zero.

$$S = \nabla \times (V H)$$

PART – B

1. ***Derive the Maxwell's equation in differential and integral form. (May / June 2016) (Nov / Dec 2015) (Apr / May 2015) (Apr / May 2014) (Apr / May 2013) (Nov / Dec 2014) (Nov / Dec 2012)***

S. No	Based on the Law	Point or Differential form	Integral form
1	Ampere's Circuital Law	$\nabla \times H = J + \frac{\partial D}{\partial t}$ OR $\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t}$	$\oint H.dl = \iint \left(J + \frac{\partial D}{\partial t}\right) ds$ OR $\oint H.dl = \iint \left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right) ds$
2	Faraday's Law	$\nabla \times E = -\frac{\partial B}{\partial t}$ OR $\nabla \times E = -\mu \frac{\partial H}{\partial t}$	$\oint E.dl = \iint \left(\frac{\partial B}{\partial t}\right) ds$ OR $\oint E.dl = -\mu \iint \frac{\partial H}{\partial t} ds$
3	Electric Gauss Law	$\nabla . D = \rho$	$\oiint D.ds = \iiint \rho\, dv$
4	Magnetic Gauss Law	$\nabla . B = 0$	$\oiint B.ds = 0$

2. ***State and explain the Poynting theorem and derive the expression for Poynting vector. (Nov / Dec 2015) (Nov / Dec 2016) (Apr / May 2015) (Apr / May 2014) (Nov / Dec 2014) (Nov / Dec 2013) (Nov / Dec 2012)***

Poynting Theorem: The vector product of electric field intensity and magnetic field intensity at any point is a measure of the rate of energy flow per unit area at that point.

Poynting Vector derivations:

From Maxwell's 1st equation and multiply it with E:

$$E.J = H.\nabla \times E - \nabla.E \times H - \varepsilon E\frac{\partial E}{\partial t}$$

Using Maxwell's 2nd equation:

$$E.J = -\frac{\mu}{2}\frac{\partial}{\partial t}H^2 - \frac{\varepsilon}{2}\frac{\partial}{\partial t}E^2 - \nabla.E \times H$$

$$\iiint E.J\, dv = \frac{\partial}{\partial t}\int_v \left(\frac{\mu}{2}H^2 + \frac{\varepsilon}{2}E^2\right) dv - \int_v E\times H.ds$$

Poynting Vector: The vector product of electric field intensity and magnetic field intensity is another product called Poynting vector.

P = E X H

3. ***Derive Maxwell's equation for time varying fields. (Nov / Dec 2016)***

S. No	Based on the Law	Point or Differential form	Integral form
1	Ampere's Circuital Law	$\nabla \times H = (\sigma + j\ \omega)E$	$\oint H.d\ = \iint (\sigma + j\ \omega)E\ d$
2	Faraday's Law	$\nabla \times E = -j\ \omega\ \mu$	$\oint E.d\ = -\mu \iint j\ \omega\ d$
3	Electric Gauss Law	$\nabla . D = \rho$	$\oiint D.d = \iiint \rho\ d$
4	Magnetic Gauss Law	$\nabla . B = 0$	$\oiint B.d = 0$

4. ***Derive the wave equation starting from Maxwell's equation for free space. (Nov / Dec 2016) (Apr / May 2014) (Nov / Dec 2014) (Nov / Dec 2013)***

For free space (dielectric medium), the ρ & σ are zero. Consider Maxwell second equation,

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t}$$

Wave equation for free space in terms of electric field: $\nabla^2 E - \mu\ \varepsilon \frac{\partial^2 E}{\partial t^2} = 0$

Consider Maxwell's First equation:

$$\nabla \times H = \varepsilon \frac{\partial E}{\partial t}$$

Wave equation for free space in terms of magnetic field: $\nabla^2 H - \mu\ \varepsilon \frac{\partial^2 H}{\partial t^2} = 0$

For free space the velocity, $v_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$.

5. ***Write Faraday's law in differential and integral forms and explain Faraday's experiment. (Apr / May 2014)***

Faraday's law: It states that the electromagnetic force (mmf) induced in a circuit is equal to the rate of decrease of the magnetic flux linking the circuit.

$$\oint E\,d\,l = -\mu \iint \frac{\partial H}{\partial t} d\,s$$

$$\oint E\,d\,l = \iint (\nabla \times E)\,d\,s$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

The electromotive force around a closed path is equal to the magnetic displacement (flux density) through that closed path.